The Pavers of Maxwell's Pathway to His Equations

Elson Agastra[1] and Stefano Selleri[2]

[1]Dept. of Electronics and Telecommunication, Faculty of Information Technology
Polytechnic University of Tirana
Sheshi "Nënë Tereza", Nr. 4, Tirana, Albania
E-mail: eagastra@fti.edu.al

[2]Dept. of Information Engineering
University of Florence
Via di Santa Marta 3, I-50134 Florence, Italy
E-mail: stefano.selleri@unifi.it

In this contribution, we will analyze and briefly present the scientists cited by Maxwell in his 1864 dissertation, "A Dynamical Theory of the Electromagnetic Field" [1]. The analysis will distinguish between researchers that are merely mentioned by Maxwell and those for which a precise citation to published papers is given. For the sake of clarity, since in [1] citations were often given in a very short form without titles or pages, the full bibliographical references are given here [2-26]. Furthermore, for these papers we will also investigate the "second-level citations," that is, researchers cited by those cited by Maxwell. A third-level investigation was done only for [27], which is a self-citation by Maxwell in [2].

It is apparent how the analysis of this citation tree originating by James Clerk Maxwell's 1864 dissertation allows us to reconstruct which previous scientific results inspired him and were synthesized in his famous equations. It also allows having a partial, yet very interesting, glimpse of the nations that were contributing the most to scientific research at that time. Of course, this glimpse, coming directly from Maxwell, expresses his personal view and conviction, at least of those he chose to mention in writing. There were surely other influences, which he may have acknowledged but choose not to mention, or even that he did not consciously recognize.

In particular, for the first level, Maxwell cited or mentioned 23 scientists, all of them European. In order of their number: seven German, six French, five English, three Italians, one Belgian, and one Irish. Portraits and very brief biographies of each of these are in the following. Furthermore, while many of these 23 scientists were known by Maxwell only through the reading of their work, with some he had direct personal knowledge or epistolary exchange (Faraday, Hockin, Jenkin [28]), showing a particular importance of these latter in developing Maxwell's view of electromagnetic phenomena. If we then descend to the second level – that is, researchers cited in the papers cited by Maxwell – we obtain the broader set of researchers reported in Table 1.

Table 1. Citations: I level refers to those researchers cited or mentioned in [1]; II level refers to those researchers cited or mentioned in papers [2-27].

Country	I Level	II Level	Total
French	6	20	26
English	5	19	24
German	7	12	19
Italian	3	5	8
Irish	1	3	4
Scottish	0*	3	3
Russian	0	2	2
Belgian	1	0	1
Dutch	0	1	1
Swiss	0	1	1

*Maxwell cites himself, but self-citations were excluded for everyone in this table.

It is important to note that nationality is not as easy to define as one may think, since boundaries in Europe have changed much in the last 150 years (Figure 1). In particular, while most European nations had more or less definite borders in 1864, Italy had just finished most of its unification process, with some regions still missing, most notably Rome and the northeast. Germany was still divided into a myriad of states of any size that would, in less than a decade (1871), be collected under the "German Empire." To assess German nationality, we hence defined here as "German" both those that were born and raised in states that are now part of Germany, and those born and raised in states that are now outside of Germany but were under strong German influence in the 19th century, such as most of Poland, up to Konigsberg, which is now in Russia.

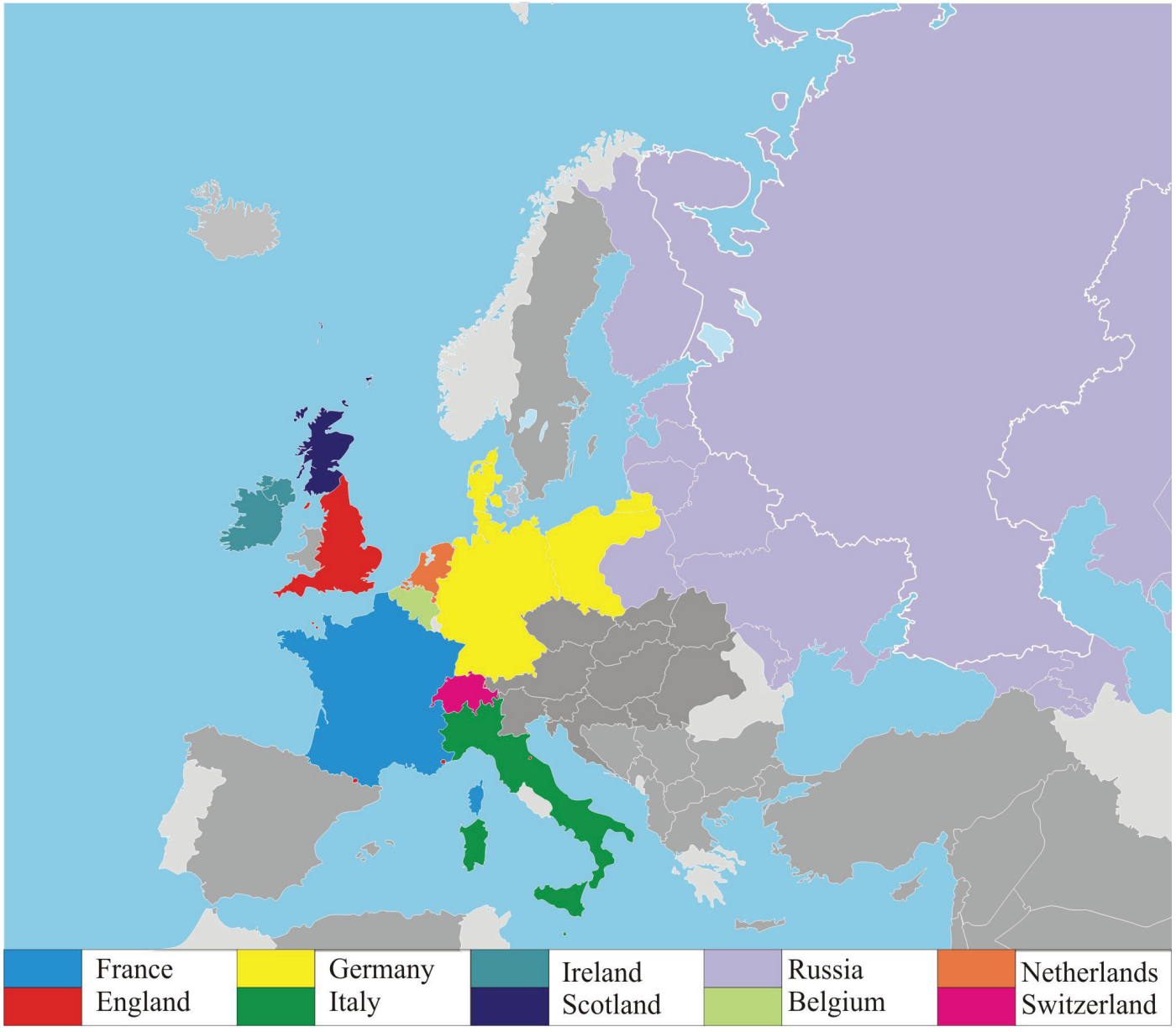


Figure 1. A political map of Europe in 1864. The colors correspond to 1864 states. The white lines within the states highlight modern boundaries. The states in color had researchers that were cited by Maxwell or within the papers he cites. All other states are in different shades of gray.

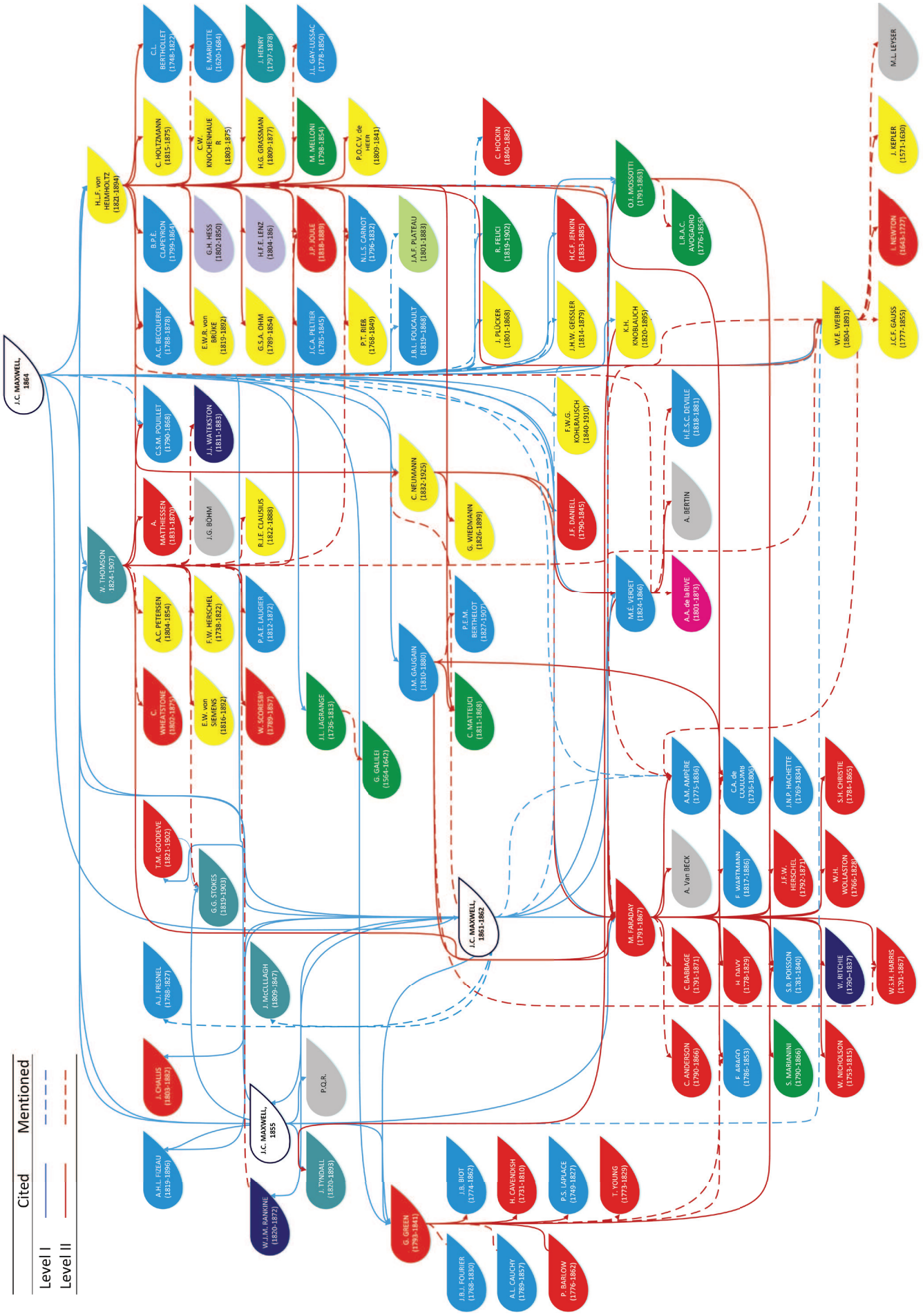


Figure 2. The citation tree for *A Dynamical Theory of the Electromagnetic Field.*

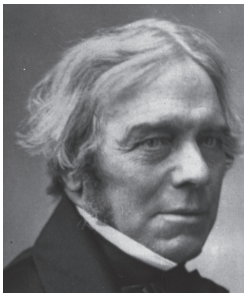
Finally, in Figure 2, the citation tree relevant to [1] is presented. Researchers are shown in boxes the colors of which match the colors of Figure 1 to show nationality, except for Henry, who was American. It was not possible to find birth and death dates and nationality for a few researchers. These are shown in gray. A possible exception is the author who signed as simply "P.Q.R," which, according to [29], could be W. Thompson, himself.

Blue arrows show citations (solid lines) and mentions (dashed lines) by Maxwell himself. Red lines show second-level citations and mentions. Maxwell is repeated three times: once for his Dynamical Theory [1], and once each for [2] and [27]. Figure 2 is quite complex and links might be not easy to follow, but the point here is actually to show how many researchers, some of which are quite obscure and forgotten, indeed directly or indirectly influenced Maxwell's paper [1], how these were formed mainly in four European countries, and how researchers even in the 19th century formed a tightly interconnected and interdisciplinary network.

Here below follows a short biography, with photo when available, of the 23 researchers cited by J. C. Maxwell.

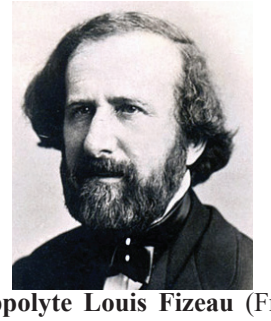
André-Marie Ampère (French, mentioned once) [Poleymieux-au-Mont-d'Or, France, January 22, 1775 – Marseille, France, June 10, 1836]. He is generally regarded as one of the main founders of the science of classical electromagnetism, which he referred to as "electrodynamics." In 1820, he knew of the discovery of Danish physicist Hans Christian Ørsted that a magnetic needle was deflected by an adjacent electric current. Ampère began developing a mathematical and physical theory to understand the relationship between electricity and magnetism. Furthering Ørsted's experimental work, Ampère showed that two parallel wires carrying electric currents attract or repel each other, depending on whether the currents flow in the same or opposite directions. This laid the foundation of electrodynamics and led to Ampère's law, which states that the mutual action of two lengths of current-carrying wire is proportional to their lengths and to the intensities of their currents. The SI (***S**ystème **I**nternational d'Unités*) unit of measurement of electric current, the ampère (symbol: A), is named after him.

John Frederic Daniell (English, mentioned twice) [London, England, March 12, 1790 – London, England, March 13, 1845]. A chemist and physicist, he was the first professor of chemistry at the newly founded King's College London (1831), and at the East India Company's Military Seminary at Addiscombe, Surrey (1835). He is best known for his invention of the Daniell cell, which was a great improvement over the Voltaic cells in electric-battery manufacturing. He also invented the dew-point hygrometer, now known by his name, and a register pyrometer. In 1830, he erected in the hall of the Royal Society a water-barometer, with which he carried out a large number of observations.



Michael Faraday (English, cited six times and mentioned three times) [Newington Butts, England, September 22, 1791 – Hampton Court, Middlesex, England, August 25, 1867]. An English scientist who contributed to the fields of electromagnetism and electrochemistry. His main discoveries included those of electromagnetic induction, diamagnetism, and electrolysis. Although Faraday received little formal education, he was one of the most influential scientists in history. It was by his research on the magnetic field around a conductor carrying a direct current that Faraday established the basis for the concept of the electromagnetic field in physics. He wrapped two insulated coils of wire around an iron ring, and found that upon passing a current through one coil a momentary current was induced in the other coil, a phenomenon now known as mutual induction. In subsequent experiments, he found that if he moved a magnet through a loop of wire, an electric current flowed in that wire. The current also flowed if the loop was moved over a stationary magnet. His demonstrations established that a changing magnetic field produced an electric field. This relation was modeled mathematically by James Clerk Maxwell as Faraday's law, which subsequently became one of the four Maxwell's equations. The SI unit of measurement of capacitance, the farad (symbol: F), is named after him.

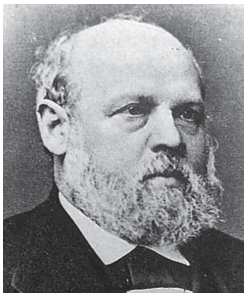
Riccardo Felici (Italian, cited once) [Parma, Italy, June 11, 1819 – Sant'Alessio, Lucca, Italy, July 20, 1902]. An Italian mathematician and physicist. Since 1851, he began studying electromagnetic induction and optics. He enunciated the law now bearing his name, which allows evaluating the total charge flowing in a circuit with induced current as the difference between the final and initial magnetic flux across the circuit, divided by the circuit's resistance [30].



Armand Hippolyte Louis Fizeau (French, cited once) (Paris, France, September 23, 1819 – Venteuil, France, September 18, 1896) A French physicist, in 1849 he calculated a value for the speed of light more precise than the previous value determined by Ole Rømer in 1676 [31]. He used a beam of light reflected from a mirror eight kilometers away. The beam passed through the gaps between the teeth of a rapidly rotating wheel. The speed of the wheel was increased until the returning light passed through the next gap and could be seen. Fizeau calculated the speed of light to be 313,300 kilometers per second, about within five percent of the correct value (299,792.458 kilometers per second).

Jean Bernard Léon Foucault (French, cited twice) [Paris, France, September 18, 1819 – Paris, France, February 11, 1868] A French physicist best known for his demonstration of the Earth's rotation via the Foucault pendulum. He also made an early measurement of the speed of light in 1850, using the Fizeau-Foucault apparatus, and showed that light travels more slowly through water than through air. In 1855, he discovered that the force required for the rotation of a copper disc becomes greater when it is made to rotate with its rim between the poles of a magnet, the disc at the same time becoming heated by the eddy current or "Foucault currents" induced in the metal.

Jean-Mothée Gaugain (French, cited once; no portrait is known to the authors) [Sully, France, 1810 – Saint-Martin-des-Entrées, France, 1880] A French engineer, in 1854 he developed an ameliorated amperometer by placing the magnetic needle at an offset with respect to the induction coil, so that the response of the needle was linear with respect to the current intensity. In 1855, he studied the rectifying action between two metal balls in an evacuated chamber, effectively producing the first electric valve.



Johann Heinrich Wilhelm Geißler (German, mentioned once) [Igelshieb, Germany, May 26, 1814 – Jena, Germany, January 24, 1879] A skilled glassblower and physicist, famous for his invention of the Geißler tube, made of glass and used as a low pressure gas-discharge tube. The Physicist Julius Plücker (see below) owed his success in the electric-discharge experiments in large measure to the instrument made by Geißler. The Geißler tube was used for entertainment throughout the 1800s, and evolved around 1910 into commercial neon lighting. Advances in Plücker and Geißler's discharge-tube technology developed into the Crookes tube, with which the electron was discovered in 1897. In 1906 it evolved into the amplifying vacuum tube, the basis of electronics and long-distance communication technologies such as radio and television.

George Green[2] (English, cited once) [Sneiton, England, July 14, 1793 – Nottingham, England, May 31, 1841] A British mathematical physicist, he wrote *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and*

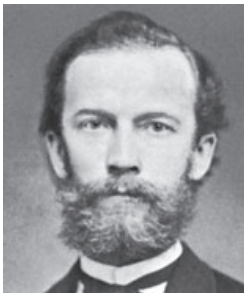
[2]A single portrait of George Green is known to the authors, the one reproduced here, but we are unsure if it really represent George Green since a recent, accurate biography of Green does not reproduce any portrait [32].

Magnetism (1828), in which he introduced several important concepts, among them a theorem similar to the modern Green's theorem, the idea of potential functions as currently used in physics, and the concept of what are now called Green's functions. Green was the first person to create a mathematical theory of electricity and magnetism. His theory formed the foundation for the work of other scientists such as James Clerk Maxwell, William Thomson, and others. His work on potential theory ran parallel to that of Carl Friedrich Gauss. Green's life story is remarkable in that he was almost entirely self-taught. He received only about one year of formal schooling as a child, between the ages of eight and nine.

Hermann Ludwig Ferdinand von Helmholtz (German, cited once) [Potsdam, Germany, August 31, 1821 – Charlottenburg, Germany, September 8, 1894] A German physician and physicist, he made significant contributions to several widely varied areas of modern science. In physics, he is known for his theories on the conservation of energy, work in electrodynamics, chemical thermodynamics, and on a mechanical foundation of thermodynamics. He became interested in electromagnetism, and the Helmholtz equation is named after him. Although he did not personally make major contributions to this field, his student, Heinrich Rudolf Hertz, became famous as the first to demonstrate electromagnetic radiation.

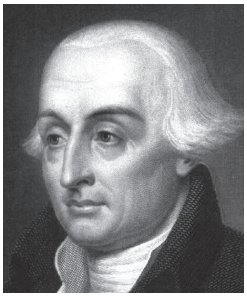
Charles Hockin (British, cited once) [1840-1882] nothing is known about him to the authors, except for the letters Maxwell exchanged with him [28].

Henry Charles Fleeming Jenkin (British, cited twice) [Dungeness, England, March 25, 1833 – Edinburgh, Scotland, June 12, 1885] He was Regius Professor of Engineering at the University of Edinburgh, and remarkable for his versatility. Known to the world as the inventor of the cable car or telpherage, he was an electrician and cable engineer, economist, lecturer, linguist, critic, actor, dramatist and artist. Jenkin joined a partnership in cable work with Varley and Thomson, whom he always admired. In 1873, Thomson and Jenkin were engineers for the Western and Brazilian cable. His treatise on *Electricity and Magnetism*, published in 1873, was notable at the time, including the latest developments in the subject. Sadly, Jenkin's papers cited by Maxwell could not be retrieved.



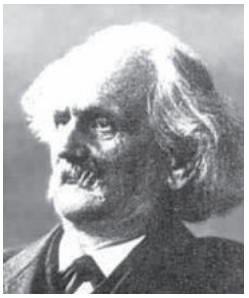
Friedrich Wilhelm Georg Kohlrausch (German, cited once, mentioned twice) [Rinteln, Germany, October 14, 1840 – Marburg, Germany, January 17, 1910] He was a German physicist who investigated the conductive properties of electrolytes and contributed to the knowledge of their behavior. He also investigated elasticity, thermoelasticity, and thermal conduction, as well as making magnetic and electrical precision measurements. Nowadays, Friedrich Kohlrausch is classed as one of the most important experimental physicists. His early work helped to extend the absolute system of Carl Friedrich Gauss and Wilhelm Weber to include electrical and magnetic measuring units.

Karl Hermann Knoblauch (German, cited once) [Berlin, Germany, April 11, 1820 – Baden Baden, Germany, June 30, 1895] A German physicist, he is most notable for his studies of radiant heat, performing valuable experiments that established some of the optical properties of radiant heat (infrared radiation). He produced valuable experimental demonstrations about the nature of diamagnetism. He also gave his time to various administrative functions in German science, including being President for 17 years of the German Academy of Sciences.



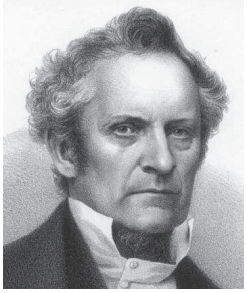
Joseph-Louis Lagrange, French; worldwide accepted version of his original Italian name: **Giuseppe Lodovico Lagrangia** (Italian, cited once) [Turin, Italy, January 25, 1736 – Paris, France, April 10, 1813] He was an Italian mathematician and astronomer who also worked for a long time in Germany and France. He made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics. In 1766, on the recommendation of Euler and d'Alembert, Lagrange succeeded Euler as the director of mathematics at the Prussian Academy of Sciences in Berlin, Prussia, where he stayed for over twenty years, producing volumes of work and winning several prizes of the French Academy of Sciences. Lagrange's treatise on analytical mechanics offered the most comprehensive treatment of classical mechanics since Newton, and formed a basis for the development of mathematical physics in the nineteenth century. In 1787, he moved from Berlin to Paris, and became a member of the French Academy. He remained in France until the end of his life. He became the first professor of analysis at the École Polytechnique upon its opening in 1794, a founding member of the Bureau des Longitudes, and Senator in 1799.

Ottaviano Fabrizio Mossotti (Italian, cited once) [Novara, Italy, April 17, 1791 – Pisa, Italy, March 20, 1863] He was an Italian physicist exiled from Italy for his liberal ideas. His name is associated with a type of multiple-element lens correcting spherical aberration and coma, but not chromatic aberration, and with the Clausius-Mossotti formula for the relationship between the dielectric constants of two different media. Mossotti was Chair of Experimental Physics in Buenos Aires (1827-1835). He taught to numerous Argentinian physicians his views on dielectrics, thereby becoming influential in the Argentine-German neurobiological tradition as regards electricity inside brain tissue. He returned to Italy, participated in military actions when in his sixties, and was later appointed Senator of Italy. Mossotti also was influential on Hendrik Lorentz's views on fundamental forces.

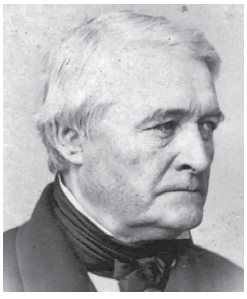


Carl Gottfried Neumann (German, cited once) [Königsberg, Prussia, (now Russia), May 7, 1832 – Leipzig, Germany, March, 1925] He was a German mathematician, son of the mineralogist, physicist, and mathematician Franz Ernst Neumann (1798-1895), who was professor of mineralogy and physics at Königsberg University. Carl Neumann studied in Königsberg and Halle, and was a professor at the universities of Halle, Basel, Tübingen, and Leipzig. Neumann worked on the Dirichlet principle, and can be considered one of the initiators of the theory of integral equations. The Neumann boundary condition for certain types of ordinary and partial differential equations is named after him.

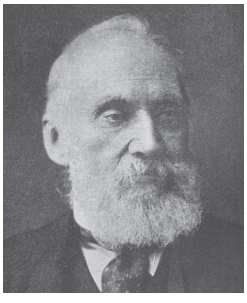
Joseph Antoine Ferdinand Plateau (Belgian, mentioned once) [Bruxelles, Belgium, October 14, 1801 – Gand, Belgium, September 15, 1883] A Belgian physicist, he was the first person to demonstrate the illusion of a moving image. To do this, he used counter-rotating disks with repeatedly drawn images in small increments of motion on one and regularly spaced slits in the other. He called this device of 1832 the phenakistoscope. Plateau also studied the phenomena of capillary action and surface tension, and the mathematical problem of the existence of a minimal surface with a given boundary is named after him. He conducted extensive studies of soap films, and formulated Plateau's laws, which describe the structures formed by such films in foams.



Julius Plücker (German, mentioned once) [Elberfeld, Germany, June 16, 1801 – Bonn, Germany, May 22, 1868] A German mathematician and physicist, he made fundamental contributions to the field of analytical geometry. He was a pioneer in the investigations of cathode rays that led eventually to the discovery of the electron. He also vastly extended the study of Lamé curves.

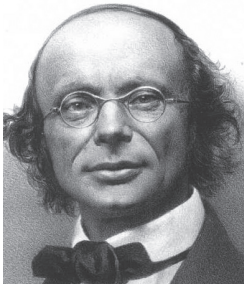


Claude Servais Mathias Pouillet (French, mentioned once) (Cusance, France, February 16, 1790 – Paris, France, June 14, 1868) He was a French physicist and a professor of Physics at the Sorbonne, and a member of the French Academy of Science. What is now known as the Pouillet effect was the phenomenon that he published in 1822 on the heat produced by the wetting of dry sand. He developed a pyrheliometer and made, between 1837 and 1838, the first quantitative measurements of the solar constant. His estimate was 1228 W/m^2 , very close to the current estimate of 1367 W/m^2 .



William Thomson, first Baron Kelvin (Irish, cited five times) [Belfast, Ireland, June 26, 1824 – Largs, Scotland, December 17, 1907] He was an Irish and British mathematical physicist and engineer. At the University of Glasgow, he did important work in the mathematical analysis of electricity, and the formulation of the first and second laws of thermodynamics. He did much to unify the emerging discipline of physics in its modern form. He also had a career as an electric telegraph engineer and inventor, which propelled him into the public eye and ensured his wealth, fame and honor. Thomson did more than any other electrician up to his time in introducing accurate methods and apparatus for measuring electricity. Acknowledging his contribution to electrical standardization, the International Electrotechnical Commission elected Thomson as its first President at its preliminary meeting, held in London on June 26-27, 1906. The SI unit of measurement of temperature, the kelvin (symbol: K) is named after him.

Marcel-Émile Verdet (French, cited twice) [Nîmes, France, March 13, 1824 – Avignon, France, June 3, 1866] A French physicist, he worked in magnetism and optics, editing the works of Augustin-Jean Fresnel. Verdet did much to champion the early theory of the conservation of energy in France through his editorial supervision of the *Annales de Chimie et de Physique*.



Wilhelm Eduard Weber (German, cited once, mentioned six times) [Wittenberg, Germany, October 24, 1804 – Göttingen, Germany, June 23, 1891] He was a German physicist and, together with Carl Friedrich Gauss, in 1833, inventor of the first electromagnetic telegraph. He studied magnetism with Gauss, and during 1864, published his *Electrodynamic Proportional Measures*, containing a system of absolute measurements for electric currents, which formed the basis of those in use. The SI unit of measurement of magnetic flux, the weber (symbol: Wb) is named after him.

References

1. J. C. Maxwell, "A Dynamical Theory of the Electromagnetic Field," *Philosophical Transactions of the Royal Society*, **155**, 1865, pp. 459-512.

2. J. C. Maxwell, "On Physical Lines of Force," *Philosophical Magazine*, ser 4, **XXI**, 1861, pp. 161-175, pp. 281-291 and pp. 338-348; ser **4**, **XXIII**, 1862, pp. 12-24 and pp. 85-95.

3. M. Faraday, *Experimental Researches in Electricity*, **III**, 1855, Series XIX, exp. 2146-2242.

4. M. Faraday, *Experimental Researches in Electricity*, **I**, 1839, Series XI, exp. 1161-1319.

5. M. Faraday, *Experimental Researches in Electricity*, **I**, 1839, Series IX, exp. 1048-1118.

6. M. Faraday, *Experimental Researches in Electricity*, **I**, 1839, Series I.3, exp. 60-80.

7. M. Faraday, *Experimental Researches in Electricity*, **III**, 1855, letter at pp. 447-452.

8. R. Felici, "Mémoire sur L'Induction Électrodynamique [Dissertation on Electrodynamic Induction]," *Annales de Chimie*, sér. 3., **XXXIV**, 1852, pp. 64-77.

9. H. Fizeau, "Sur une expérience relative à la vitesse de propagation de la lumière [On an Experiment on the Speed of Light]," *Comptes Rendus*, **XXIX**, 1849, p. 90-92 and 132.

10. L. Foucault, "Détermination expérimentale de la vitesse de la lumière; parallaxe du Soleil [Experimental Determination of Light Speed, Solar Parallax]," *Comptes Rendus*, **LV**, 1862, pp. 501-503 and 792-800.

11. J.-M. Gaugain, "Mémoire sur la conducibilité électrique et la capacité inductive des corps isolants," *Annales de Chimie et Physique*, ser. 4, **II**, 1864, pp. 264-316.

12. G. Green, *An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*, Weelhouse, Nottingham, UK, 1828.

13. H. Helmholtz, *Über die Erhaltung der Kraft* [Conservation of Forces], Physical Society of Berlin, Druck und Verlag von G. Reimer, Berlin (D), 1847.

14. W. Weber and R. Kohlraush, "Über die Elektricitätsmenge, welche bei galvanishen Strömen durch den Quershnitt der Kette fliefst [On the Amount of Electricity Which Flows in Galvanic Currents Through the Chain Cross Section]," *Annalen der Physik und Chemie*, ser. 2, **XCIX**, 1856, pp. 10-25.

15. K. H. Knoblauch, "On the Deportment of Crystalline Bodies Between the Electric Poles," *Philosophical Magazine*, ser. 4, **II**, 1851, pp. 33-36.

16. J.-L. Lagrange, *Méchanique Analitique*, Veuve Desaint, Paris, 1788.

17. O. F. Mossotti, "Discussione analitica sull'influenza che l'azione di un mezzo dielettrico ha sulla distribuzione dell'elettricità alla superficie di più corpi elettrici disseminati in esso [Analytical discussion on the influence of a dielectric body on the electrical distribution on the surface of conductors within the body itself]," *Memorie di Matematica e Fisica della Società Italiana delle Scienze in Modena*, **XXIV**, pp. 1850

18. C. Neumann, *Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur* [A tentative explanation on how the linearly polarized light is depolarized by electric or magnetic forces], Shmidt, Halis Saxonum (Halle, D), 1858.

19. W. Thompson, "On the Possible Density of the Luminiferous Medium, and on the Mechanical Value of a Cubic Mile of Sunlight," *Transactions of the Royal Society of Edinburgh*, **XXI**, (read 1854, published 1857), pp. 57-61.

20. W. Thompson, "Dynamical Illustrations of the Magnetic and the Helissoidal Rotatory Effects of Transparent Bodies on Polarized Light," *Proceedings of the Royal Society*, **VIII**, (received May 1856 printed in 1857), pp. 150-158.

21. W. Thompson, "On the Measurement of Electric Resistance," *Proceedings of the Royal Society*, **XI**, (received June 1861 printed 1862), pp. 313-328.

22. W. Thompson, "On the Mechanical Theory of Electrolysis," *Philosophical Magazine*, ser. 4, **II**, 1851, pp. 429-444.

23. W. Thompson, "On the Mechanical Energies of the Solar System," *Transactions of the Royal Society of Edinburgh*, **XXI**, (read 1854, published 1857), pp. 63-80.

24. E. Verdet, "Note sur les propriétés optiques des corps transparents soumis à l'action du magnétisme [A note on the optical properties of transparent bodies under the effect of magnetic fields]," *Comptes Rendus*, **XLIII**, 1856, pp. 529-533.

25. E. Verdet, "Note sur les propriétés optiques des corps magnetiques [A note on the optical properties of magnetic bodies]," *Comptes Rendus*, **XLIV**, 1857, pp. 1209-1213.

26. W. Weber, "Electrodynamische Maassbestimmungen [On the Measurement of Electro-dynamic Forces]," *ANnalen der Physik et Chemie*, ser. 2, **LXXIII**, 1848, pp. 193-240.

27. J.C. Maxwell, "On Faraday's Lines of Force," *Transactions of the Cambridge Philosophical Society*, **X**, [Lecture given December 10, 1855 and February 11, 1866, Volume published 1864], pp. 27-83.

28. L. Campbell and W. Garnett, *The Life of James Clerk Maxwell*, London, UK, McMillan and Co., 1882.

29. S. P. Thompson, *The life of William Thompson, Baron Kelvin of Largos*, London, McMillan and Co., 1910.

30. E. Agastra, S. Selleri, "J. C. Maxwell's Forerunners: Riccardo Felici," *IEEE Antennas Propagation Magazine*, **54**, 3, June 2012, pp. 250-257.

31. G. Pelosi, S. Selleri, "'c' utrum est ut Celeritas an Constantia? (Does 'c' Stand for Speed or Constancy?)," *IEEE Antennas and Propagation Magazine*, **52**, 6, December 2010, pp. 207-219.

32. D. M. Cannell, *George Green, Mathematician and Physicist 1793-1841, The Background to his Life and Work, Second Edition*, London and Atlantic Highlands, NJ, SIAM, 2001.